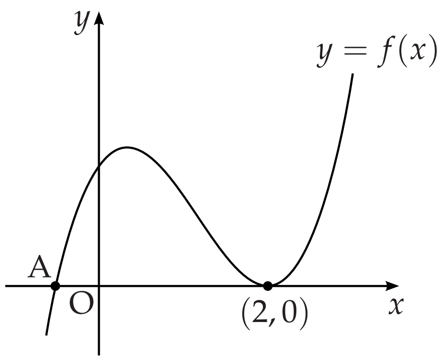
remainder theorm

[SQA] 1. (a) Given that $x+2$ is a factor of $2x^3 + x^2 + kx + 2$, find the value of k. **3**

(b) Hence solve the equation $2x^3 + x^2 + kx + 2 = 0$ when k takes this value. **2**

[SQA] 2. The diagram shows part of the graph of the curve with equation $y = 2x^3 - 7x^2 + 4x + 4$.



(a) Find the x-coordinate of the maximum turning point. **5**

(b) Factorise $2x^3 - 7x^2 + 4x + 4$. **3**

(c) State the coordinates of the point A and hence find the values of x for which $2x^3 - 7x^2 + 4x + 4 < 0$. **2**

3. Functions f, g and h are defined on the set of real numbers by

- $f(x) = x^3 - 1$
- $g(x) = 3x + 1$
- $h(x) = 4x - 5$.

(a) Find $g(f(x))$. **2**

(b) Show that $g(f(x)) + xh(x) = 3x^3 + 4x^2 - 5x - 2$. **1**

(c) (i) Show that $(x-1)$ is a factor of $3x^3 + 4x^2 - 5x - 2$.

(ii) Factorise $3x^3 + 4x^2 - 5x - 2$ fully. **5**

(d) Hence solve $g(f(x)) + xh(x) = 0$. **1**

4. (a) (i) Show that $(x-1)$ is a factor of $f(x) = 2x^3 + x^2 - 8x + 5$.

(ii) Hence factorise $f(x)$ fully. **5**

(b) Solve $2x^3 + x^2 - 8x + 5 = 0$. **1**

(c) The line with equation $y = 2x - 3$ is a tangent to the curve with equation $y = 2x^3 + x^2 - 6x + 2$ at the point G.

Find the coordinates of G. **5**

(d) This tangent meets the curve again at the point H.

Write down the coordinates of H. **1**

[SQA] 5. Factorise fully $2x^3 + 5x^2 - 4x - 3$. **4**

[SQA] 6. (a) Show that $x = 2$ is a root of the equation $2x^3 + x^2 - 13x + 6 = 0$. **1**

(b) Hence find the other roots. **3**

[SQA] 7. Find p if $(x + 3)$ is a factor of $x^3 - x^2 + px + 15$. **3**

[SQA] 8. When $f(x) = 2x^4 - x^3 + px^2 + qx + 12$ is divided by $(x - 2)$, the remainder is 114.

One factor of $f(x)$ is $(x + 1)$.

Find the values of p and q. **5**

[SQA] 9. Find k if $x - 2$ is a factor of $x^3 + kx^2 - 4x - 12$. **3**

[SQA] 10. One root of the equation $2x^3 - 3x^2 + px + 30 = 0$ is -3.

Find the value of p and the other roots. **4**

[SQA] 11. (a) Show that $(x - 3)$ is a factor of $f(x)$ where $f(x) = 2x^3 + 3x^2 - 23x - 12$. **2**

(b) Hence express $f(x)$ in its fully factorised form. **2**

[SQA] 12. Express $x^4 - x$ in its fully factorised form. **4**

[SQA] 13. (a) Find a real root of the equation $2x^3 - 3x^2 + 2x - 8 = 0$. **2**

(b) Show algebraically that there are no other real roots. **3**

[SQA] 14. Express $x^3 - 4x^2 - 7x + 10$ in its fully factorised form. **4**

[SQA] 15.

(*a*) The function f is defined by $f(x) = x^3 - 2x^2 - 5x + 6$.

The function g is defined by $g(x) = x - 1$.

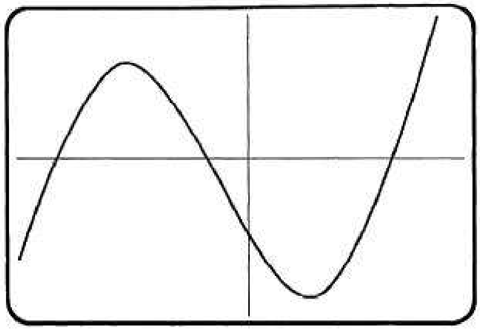
Show that $f\big(g(x)\big) = x^3 - 5x^2 + 2x + 8$. **4**

(*b*) Factorise fully $f\big(g(x)\big)$. **3**

(*c*) The function k is such that $k(x) = \dfrac{1}{f\big(g(x)\big)}$.

For what values of x is the function k not defined? **3**

[SQA] 16. The diagram shows part of the graph of the curve with equation $f(x) = x^3 + x^2 - 16x - 16$.



(*a*) Factorise $f(x)$. **(3)**

(*b*) Write down the co-ordinates of the four points where the curve crosses the x and y axes. **(2)**

(*c*) Find the turning points and justify their nature. **(6)**

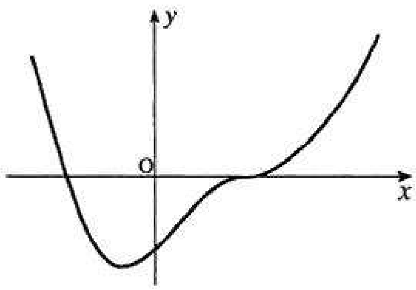
[SQA] 17. The graph of the curve with equation $y = 2x^3 + x^2 - 13x + a$ crosses the x-axis at the point (2,0).

(*a*) Find the value of a and hence write down the coordinates of the point at which this curve crosses the y-axis. **(3)**

(*b*) Find algebraically the coordinates of the other points at which the curve crosses the x-axis. **(4)**

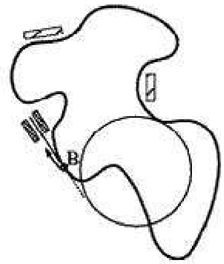
[SQA] 18. The function f, whose incomplete graph is shown in the diagram, is defined by $f(x) = x^4 - 2x^3 + 2x - 1$.

Find the coordinates of the stationary points and justify their nature.

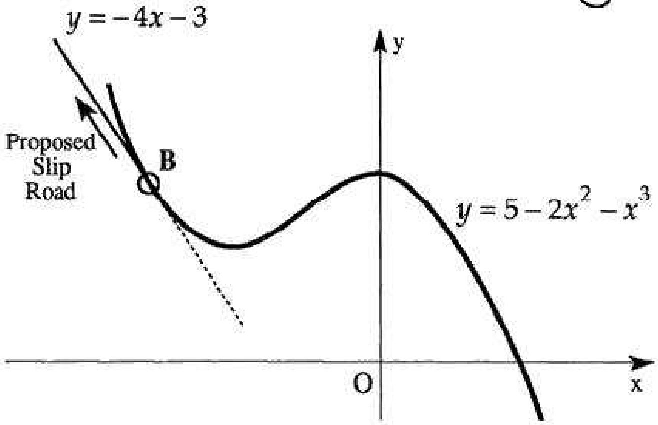


(8)

[SQA] 19. The diagram shows the plans for a proposed new racing circuit. The designer wishes to introduce a slip road at B for cars wishing to exit from the circuit to go into the pits. The designer needs to ensure that the two sections of road touch at B in order that drivers may drive straight on when they leave the circuit.



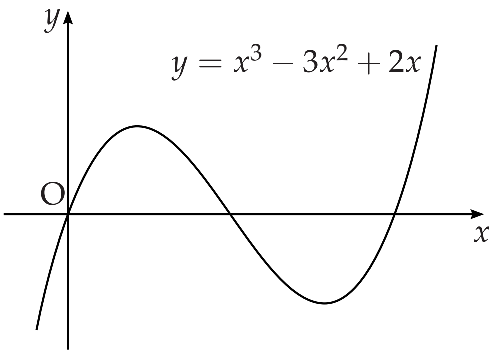
Relative to appropriate axes, the part of the circuit circled above is shown below. This part of the circuit is represented by a curve with equation $y = 5 - 2x^2 - x^3$ and the proposed slip road is represented by a straight line with equation $y = -4x - 3$.



(*a*) Calculate the coordinates of B. **(7)**

(*b*) Justify the designer's decision that this direction for the slip road does allow drivers to go straight on. **(1)**

[SQA] 20. The diagram shows a sketch of the graph of $y = x^3 - 3x^2 + 2x$.



(*a*) Find the equation of the tangent to this curve at the point where $x = 1$. **5**

(*b*) The tangent at the point $(2, 0)$ has equation $y = 2x - 4$. Find the coordinates of the point where this tangent meets the curve again. **5**

[SQA] 21.

(*a*) (i) Show that $x = 1$ is a root of $x^3 + 8x^2 + 11x - 20 = 0$.

(ii) Hence factorise $x^3 + 8x^2 + 11x - 20$ fully. **4**

(*b*) Solve $\log_2(x + 3) + \log_2(x^2 + 5x - 4) = 3$. **5**

[END OF QUESTIONS]